



Innovative and Creative ideas of Mr. Mohammadreza Akbari in Analytically Solving Nonlinear Differential Equations in Engineering Fields and Challenging Existing Methods in the World

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Abstract

In this article, we want to challenge many difficult scientific problems in engineering and physics with the creative method of Mr. Mohammad Reza Akbari, which many scientists have difficulty solving or have not been able to solve many such nonlinear problems analytically. In fact, this article is an exponential power against other old methods for the analytical solution of nonlinear differential equations, as well as introducing these innovations to students around the world who can use these creative ideas in the design of difficult engineering problems in the near all scientists know, the behavior of most phenomena (God's creations in the universe) in practical projects are non-linear, and these phenomena follow non-linear partial differential equations, the solution of which is a problem for scientists. it can be said that the analytical solution of many such differential equations is impossible and is a challenge for engineering and physics research are many non-linear differential equations in various fields of engineering and physics that remain unsolved and few scientists have been able to analyze some of them (a list of such equations can be found on Google). In this article, we want to prove that it is possible to solve any type of very complex nonlinear differential equation with the new innovative methods of Mr. Mohammad Reza Akbari and to present this kind of applied equations to the world through analytical solution, that these new innovations of Mr. Mohammadreza Akbari can move the boundaries of non-linear sciences, and these ideas can be easily and flexibly available to scientists in various fields of engineering and basic example, until now, no method in the world has been able to solve nonlinear differential equations in terms of constant coefficients (C_1, C_2, \dots) (that is, without boundary conditions or initial conditions). As a result, one of the "beauty of analytical solution" for nonlinear differentials is the general solution of differential equations (without initial and boundary conditions). One of the biggest problems of solving old methods (such as HPM, ADM, VIM, DTM) is in this regard. Mohammadreza Akbari's new creative and innovative methods can provide a "general solution" to all kinds of complex nonlinear differential equations.

Keywords: New Approaches, Innovations of M R Akbari, Nonlinear D.E (ode, pde), Analytical Solutions, General Solution, Integral.

Introduction

In this article, the power and efficiency of Mr. Mohammad Reza Akbari's innovative methods are shown compared to the existing methods in academic and research societies.

In this article, we want to prove that with innovative and flexible ideas and methods, it is possible to create change and transformation in the field of nonlinear differential equations for the world. Also, the use of innovative ideas can solve the concerns of researchers and scientists in the analytical solution of nonlinear differential equations. Everyone knows that in international

scientific competitions, innovative solutions and ideas that first have high accuracy, secondly have a simple algorithm and flexibility in analyzing non-linear problems, and thirdly are practical, have usually won the field. As a result of presenting this article, with the creative solutions Mr. Mohammad Reza Akbari has devised from 2014 to 2024, they can easily meet the scientific needs of researchers. And I hope that these powerful new methods that I have created will not be put to unconventional use for users in the world and will be used in a peaceful way. The innovative methods I have created are called with abbreviation 1-15.

Mathematical Formulation of The Problem

We consider a few complicated examples in nonlinear differential equations (Pde and Ode) and integral to represent engineering field and basic science problems.

i) Analytical solution of nonlinear partial differential equations (PDE) by MrAM method (Mohammadreza Akbari Method)

Example 1

We consider a Vibration **Dynamic nonlinear of partial differential equation** as a nonlinear phenomenon in the engineering field and basic sciences as follows:

$$\frac{\partial^2 u}{\partial t^2} + \alpha \frac{\partial^4 u}{\partial x^4} + \varepsilon \frac{\partial^2 u}{\partial x^2} + \beta u^4 = 0 \quad (1)$$

The boundary and initial conditions are:

$$\begin{aligned} u(0, t) = 0, u_x(0, t) = 0, u_x(L, t) = 0 \\ u_{xx}(0, t) = 0, u(x, 0) = A, u_t(x, 0) = B \end{aligned} \quad (2)$$

Mr AM Solution Process (Mohammadreza Akbari Method)

Output of the solution process by new approach Mr AM (Mohammadreza Akbari Method) for nonlinear differential equation Eq.(1) and according to the boundaries and initials conditions Eq.(2), the solution of the differential equation is obtained as follows:

$$u(x, t) = -\phi \left\{ \alpha \frac{\partial^4 \psi}{\partial x^4} + \varepsilon \frac{\partial^2 \psi}{\partial x^2} + \beta \psi^4 \right\} + \psi \quad (3)$$

The analytical solution of Eq.(3) in the MrAM method is obtained in terms of **time** and **space** functions as follows:

$$\psi = -Z(t)\phi ; \phi := \frac{1}{2}x^2(3L - 2x)(L - x) \quad (4)$$

$$\begin{aligned} Z(t) = \frac{189A}{19L^4} \cos \left\{ \frac{1}{2888L^2} \right. \\ \left. \times \sqrt{1.2 \times 10^7 A^3 L^4 \beta - 94818816L^2 \varepsilon + 1991195136\alpha t} \right\} \end{aligned} \quad (5)$$

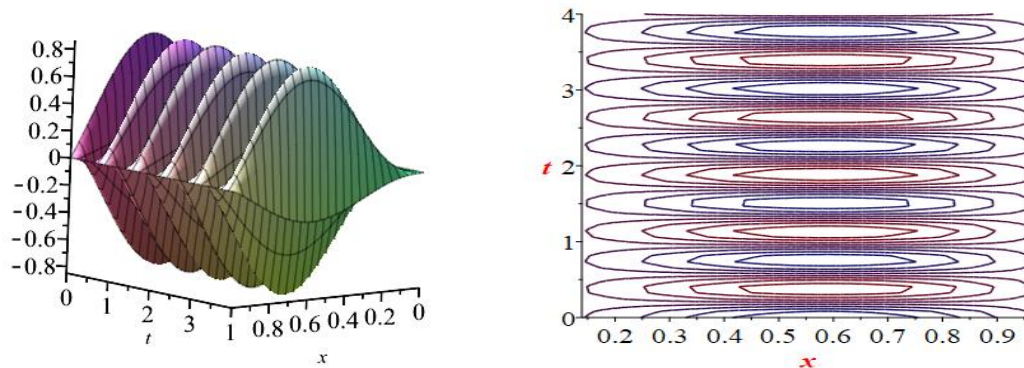
The following values have been used for the physical parameters of this problem of Eqs.(3-5)as:

$$\alpha := 0.3; \beta := 0.1; \varepsilon := 0.2; L := 1; A := 0.5; B := 0 \quad (6)$$

By substituting the physical values Eq.(6) in the resulting Eqs.(3-5) as:

$$\begin{aligned} u(x, t) = \frac{1}{2} x^2 (3 - 2x) (1 - x) \times \\ \left\{ -3.825 \cos(8.3286 t) x^8 (3 - 2x)^4 (1 - x)^4 \right. \\ \left. - 0.995 \cos(8.33 t) (3 - 2x) (1 - x) + 3.99 \cos(8.33 t) x (1 - x) \right. \\ \left. + 1.99 \cos(8.33 t) x (3 - 2x) - 1.99 \cos(8.33 t) x^2 \right\} \\ + 2.487 \cos(8.33 t) x^2 (3 - 2x) (1 - x) \end{aligned} \quad (7)$$

Graphs of Tet equation in 3D coordinates as well as contour graphs shown below:



Figs.1. 3D charts and contour plots by MrAM and Numerical solution.

The time diagram of Eq.(7) is shown below in time $t=2$ sec as:

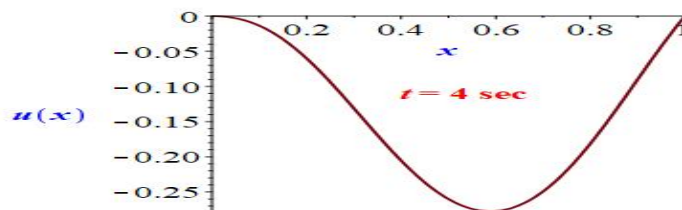


Figure 2: Time chart for $t=2$ sec.

We compare solution of Mr AM method of Eq.(5) with the numerical method (Runge-Kutte 4th) , as shown below:

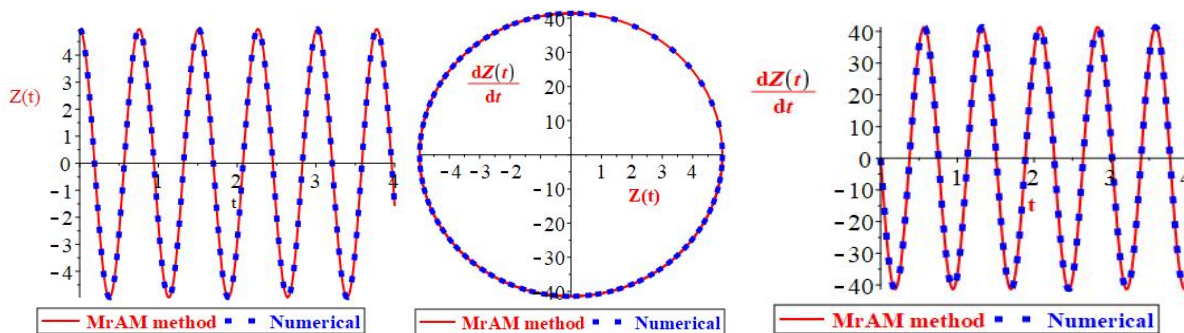


Figure 3: A comparison between MrAM and Numerical solution.

Example 2

Consider the following 3-dimensional nonlinear sixth-order PDE an we solve by AYM method:

$$\frac{\partial u}{\partial t} = \alpha \cdot \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \eta \left(\frac{\partial^6 u}{\partial x^6} + \frac{\partial^6 u}{\partial y^6} + \frac{\partial^6 u}{\partial z^6} \right) + \beta \left(\frac{\partial^4 u}{\partial x^4} + \frac{\partial^4 u}{\partial y^4} + \frac{\partial^4 u}{\partial z^4} \right) + \mu u^2, \quad u = u(x, y, z, t) \quad 1$$

Where the boundary and initial conditions can be expressed as:

$$\begin{aligned} u(0, y, z, t) = 0, u(a, y, z, t) = 0, u_{xx}(0, y, z, t) = 0, u_{xx}(a, y, z, t) = 0 \\ u(x, 0, z, t) = 0, u(x, b, z, t) = 0, u_{yy}(x, 0, z, t) = 0, u_{yy}(x, b, z, t) = 0 \\ u(x, y, 0, t) = 0, u(x, y, c, t) = 0, u_{yy}(x, y, 0, t) = 0, u_{yy}(x, y, c, t) = 0 \\ u(x, y, z, 0) = u_0 \end{aligned} \quad 2$$

Using the **AYM** method and based on the eigenvalue function of $\phi(x, y, z) = A \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi y}{b}\right) \sin\left(\frac{p\pi z}{c}\right)$, an analytical solution of eq.(1) can be obtained as:

$$u(x, y, z, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sum_{p=1}^{\infty} \left(A \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi y}{b}\right) \sin\left(\frac{p\pi z}{c}\right) \times \exp\left[\frac{-\psi \pi t}{c^6 b^6 a^6} - \text{LambertW}\left(\frac{64 A \mu t}{27 n m p \pi^3} e^{-\frac{\pi^2 \psi t}{c^6 b^6 a^6}} \right) \right] \right) \quad 3$$

Where the parameter A, ψ and the rest of the physical parameters involved in the presented solution are defined as follows:

$$A = \frac{8 u_0}{\pi^3 n p m} [1 - (-1)^m - (-1)^n - (-1)^p + (-1)^{m+n} + (-1)^{m+p} + (-1)^{n+p} - (-1)^{m+n+p}] \quad 4$$

$$\psi = \pi^4 a^6 b^6 \eta p^6 + \pi^4 a^6 c^6 \eta m^6 + n^6 \eta b^6 c^6 \pi^4 - \pi^2 a^6 b^6 \beta c^2 p^4 - \pi^2 a^6 b^2 \beta c^6 m^4 - n^4 \beta a^2 b^6 c^6 \pi^2 \quad 5$$

$$+ a^6 \alpha b^6 c^4 p^2 + a^6 \alpha b^4 c^6 m^2 + n^2 \alpha a^4 b^6 c^6 \quad 6$$

$a = 1, b = 2, c = 1, \alpha = -0.9, \beta = 0.1, \mu = 0.01, \eta = 0.02, u_0 = 1$

Figure 2.13 shows the variation of the obtained analytical solution at different time values for $u(x)$, $u(y)$ and $u(z)$ in the following form:

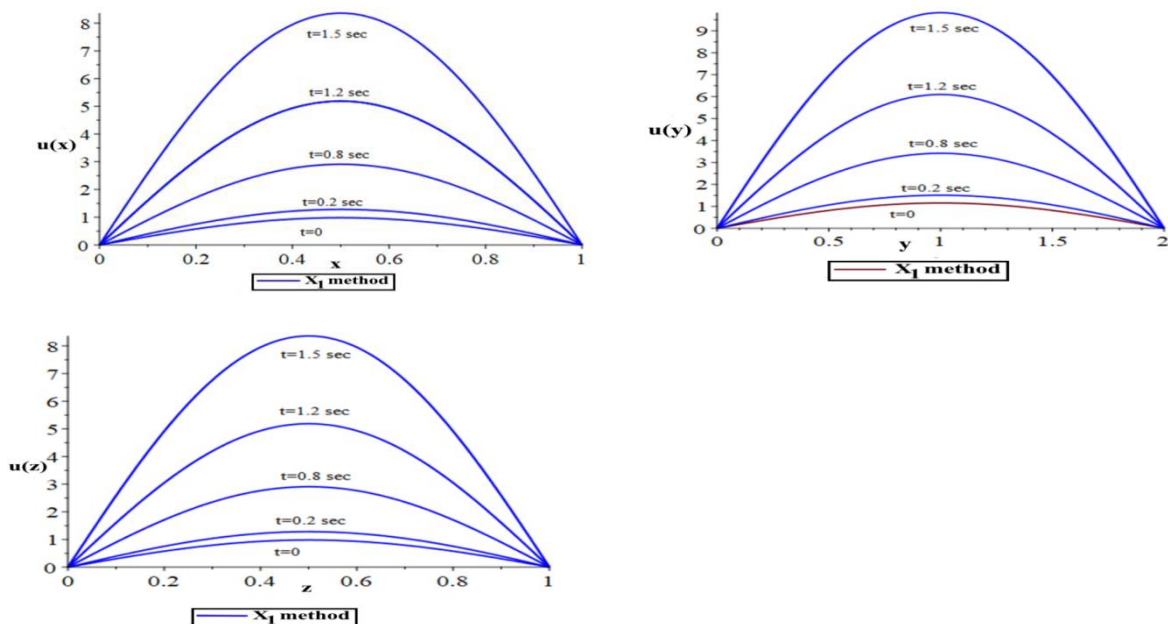


Figure 1: The effect of different time values on the obtained analytical solution for $u(x)$ at $y=0.6, z=0.2$ and for $u(y)$ at $x=0.2$ and $z=0.6$ and for $u(z)$ at $x=0.2$ and $y=0.6$ by **AYM** method.

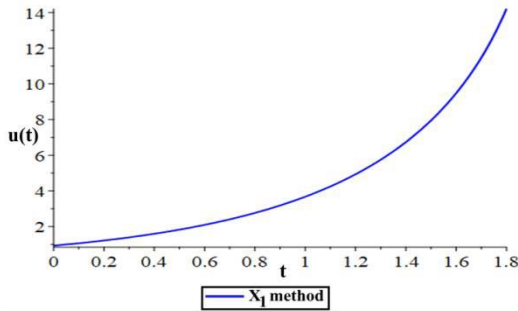


Figure 2.14: Time evolution of the obtained analytical solution as a function of time using $u(t)$ at $x=0.2, y=0.6$, and $z=0.6$.

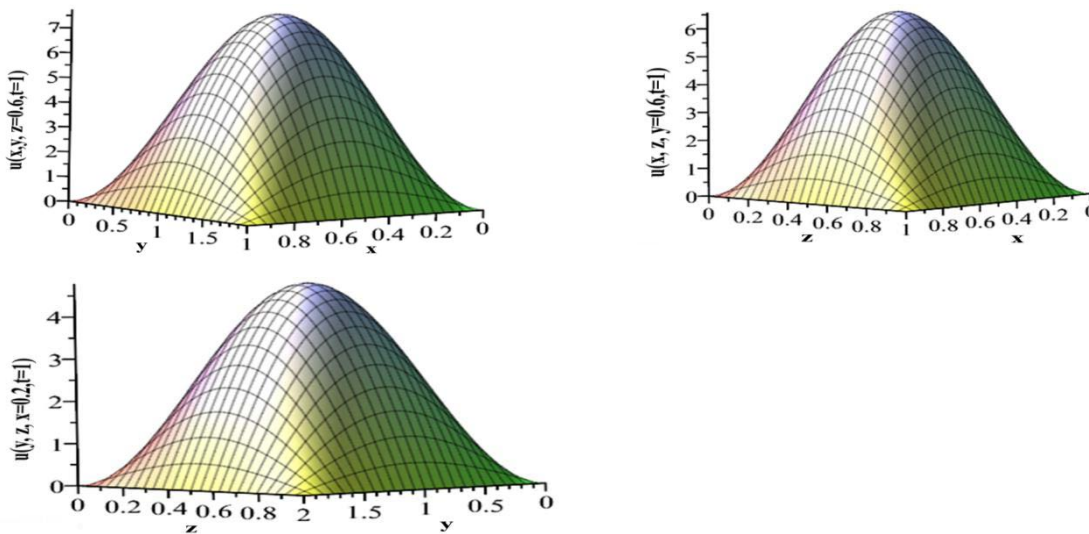


Figure 2.15: The 3D charts resulted from the presented problem's analytical solution using the AYM method. The functions $u(x,y)$, $u(x,z)$ and $u(y,z)$ have been depicted at $t=1$ sec.

Example 3

We consider a **nonlinear partial differential equation** for **Heat transfer** as a nonlinear phenomenon in the engineering field and basic sciences in the coordinates of the **Spherical** as follows:

$$\frac{\partial u}{\partial t} + \frac{\alpha}{x^2} \frac{\partial}{\partial x} \left(x^2 u \frac{\partial u}{\partial x} \right) + \epsilon u^2 + \eta u \frac{\partial u}{\partial x} = 0 \quad (1)$$

The boundary and initial conditions are:

$$u_x(0, t) = u1, u(L, t) = u2, u(x, 0) = u0 \quad (2)$$

Mr AM solution process (Mohammadreza Akbari Method)

Output of the solution process by new approach **Mr AM (Mohammadreza Akbari Method)** for nonlinear differential equation Eq.(1) and according to the boundaries and initials conditions Eq.(2), the solution of the differential equation is obtained as follows:

$$u(x, t) = \phi f(\psi) + \psi \quad (3)$$

The analytical solution of Eq.(3) in the Mr AM method is obtained in terms of **time** and **space** functions as follows:

$$\psi := (x^2 - L^2) Z(t) + u_2 - u_1(L - x) \quad (4)$$

$$f(\psi) = \frac{-1}{\alpha} \left(\frac{2}{x} \left(\frac{\partial \psi}{\partial x} \right) + \frac{1}{\psi} \left(\frac{\partial \psi}{\partial x} \right)^2 + \frac{\varepsilon \psi}{\alpha} + \psi \frac{\partial \psi}{\partial x} \right) \quad (5)$$

$$Z(t) = \frac{5}{\Gamma} \left\{ 224L^2\varepsilon - 105\eta L + \sqrt{\Delta} \tan \left[\frac{1}{112L^2} \left\{ 112L^2 \arctan \left(\frac{2}{\sqrt{\Delta}} (8L^2\varepsilon - 35\eta L - 280\alpha) \right) + 5t\sqrt{\Delta} \right\} - 840\alpha \right] \right\} \quad (6)$$

$$\Delta = 3584L^4\varepsilon^2 + 7840L^3\eta\varepsilon + 62720L^2\alpha\varepsilon - 11025L^2\eta^2 - 176400L\alpha\eta - 705600\alpha^2 \quad (7)$$

$$\Gamma = 2L^2(48L^2\varepsilon - 35\eta L - 280\alpha) \quad (8)$$

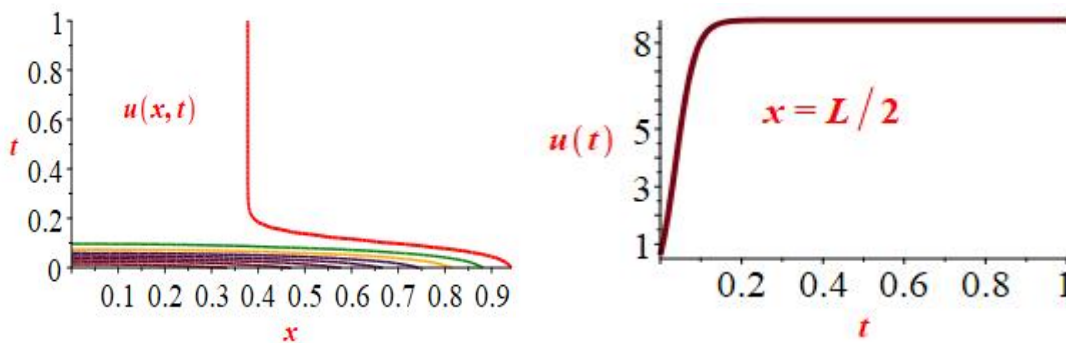
The following values have been used for the physical parameters of this problem of Eqs.(3-8)as:

$$\alpha := -0.5; \varepsilon := 0.6; L := 1; \eta := -0.2; u_1 := 0; u_2 := 10; u_0 := 20 \quad (9)$$

By substituting the physical values Eq.(9) in the resulting Eqs.(4-8) as:

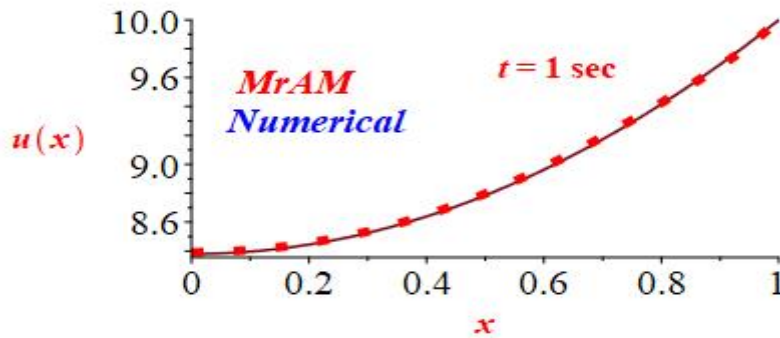
$$u(x, t) = 1.817 + 8.1826x^2 - 6.56232i \tan(-0.789i + 20.601it) + 6.5623i \tan(-0.789i + 20.601it)x^2 \quad (10)$$

Graphs of time u(t) equation in 3D coordinates as well as contour graphs shown below:



Figs.4. contour plots by MrAM and time function u(t).

We compare solution of **Mr AM** method of Eq.(6) with the numerical method (Runge-Kutte 4th) for $t=1\text{sec}$, as shown below:



Figs.5.A comparison between Mr AM and Numerical solution for $t=1$ sec.

ii) Analytical solution nonlinear differential equations for Fractional Calculus by ‘I am or IAM’ method (Integral Akbari Method)

Example 1

We consider 2 nonlinear differential equations (ODE , PDE) for **Fractional Calculus** as follows:

$$1. \text{ ode : } \frac{d^{1/2}u}{dx^{1/2}} = \mu u \frac{du}{du} + \varepsilon \sin(\beta u) \quad (1)$$

And boundary conditions as:

$$bc : u(0) = u1 , u(L) = u2 \quad (2)$$

$$2. \text{ pde : } \frac{d^{1/2}u}{dt^{1/2}} = \beta \frac{\partial^2 u}{\partial x^2} + \varepsilon \frac{du}{dx} + \eta u^2 \quad (3)$$

And boundary conditions as:

$$bc, ic : u(0, t) = u1 , u(L, t) = u2 , u(x, 0) = uo \quad (4)$$

1.ode: The answer nonlinear differential equation by **IAM** as follows:

$$u(x) = 1 - \frac{\sqrt{x}}{6 \pi^{5/2}} \left\{ \varepsilon \beta \pi \cos \frac{\beta x}{2 \pi} \cos \beta \right. \\ + \varepsilon \beta \pi \sin \frac{\beta x}{2 \pi} \sin \beta + 2 \varepsilon \beta \pi \cos \left(\frac{\beta}{9 \pi} (9 \pi - 4 x) \right) \\ + 2 \varepsilon \beta \pi \cos \left(\frac{\beta}{18 \pi} (18 \pi - 5 x) \right) + \varepsilon \beta \pi \cos \beta - 3 \mu \left. \right\} \\ + \{ 0.012 \beta \varepsilon \cos^2 \beta + 0.012 \beta \varepsilon \sin^2 \beta \\ + 0.024 \beta \varepsilon \cos(0.12 \beta) + 0.024 \beta \varepsilon \cos(0.45 \beta) \\ + 0.012 \beta \varepsilon \cos(\beta) - 0.0114 \mu - 0.16 \} x \quad (5)$$

By selecting physical values as:

$$\beta = 3 , \varepsilon = -0.8 , \mu = 1.5 , L = 2\pi \quad (6)$$

The obtained analytical solution can be compared with numerical results, as shown:

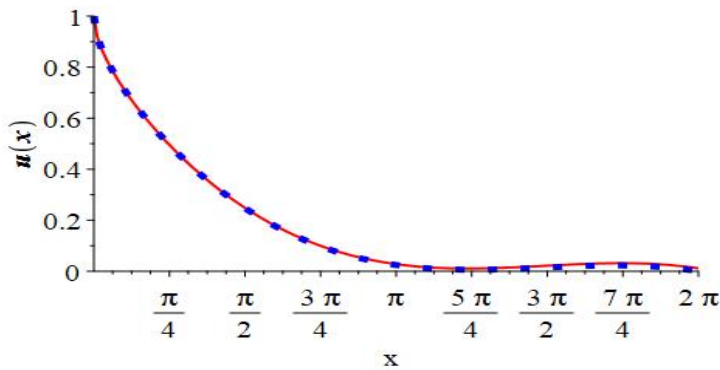


Fig.1. A comparison between IAM and Numerical solution.

2.pde: The answer nonlinear differential equation by IAM as follows:

$$\begin{aligned}
 u(x, t) = & \frac{\sqrt{x}}{2\sqrt{\pi}} \left\{ \lambda [8\eta\pi - 4\beta\pi^3 - 6\eta\pi x] \cos\frac{3\pi x}{4} \right. \\
 & + \lambda [8\eta\pi\lambda - 4\epsilon\pi^2 - 8\eta] \sin\frac{3\pi x}{4} + \lambda [4\pi\eta - 4\pi\eta x \\
 & - 2\beta\pi^3] \cos(\pi x) - \lambda [2\pi^2\epsilon + 4\eta] \sin(\pi x) \\
 & + 4\pi\eta\lambda^2 \sin(2\pi x) - 2\beta\lambda\pi^3 + 4\pi\eta\lambda + 5\eta x - 8\eta \left. \right\} \\
 & - (24.74\beta\lambda - 7.875\epsilon\lambda + 0.7\eta\lambda - 7.09\eta\lambda^2 + 1 \\
 & - 0.8463\eta) x + 1 \quad , \quad \lambda = e^{-\frac{\beta\pi^2 t}{L^2}}
 \end{aligned} \tag{7}$$

By selecting physical values as:

$$\beta = 0.7, \eta = 0.2, \epsilon = 0.1, u_1 = 1, u_2 = 0, u_0 = 1, L = 1$$

The Obtained Analytical Solution Can Be Compared with Numerical Results, As Shown

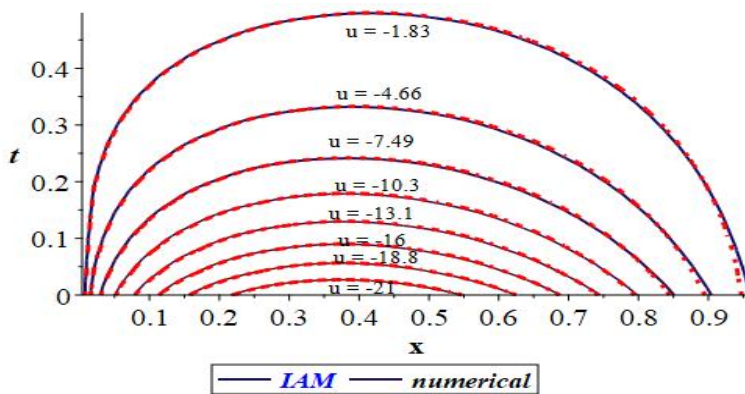


Fig.2. A comparison between IAM and Numerical solution.

iii) Analytical solution of vibrational nonlinear differential equations (ODE) by LAM method (Linearize Akbari Method)

Example 1

We Consider a Vibration Nonlinear Differential Equation as A Nonlinear Phenomenon in The Engineering Field and Basic Sciences as Follows:

$$\ddot{u} + \sin(\alpha u) + \beta u^p + \mu \dot{u} = 0 \quad (1)$$

The boundary and initial conditions are:

$$u(0) = A, u'(0) = B \quad (2)$$

LAM Solution Process (Linearize Akbari Method)

Output of the solution process by new approach LAM (Linearize Akbari Method) for nonlinear differential equation Eq.(1) and according to the boundaries and initials conditions Eq.(2), the solution of the differential equation is obtained as follows:

$$u(t) = \frac{1}{36p^p i \omega} \left\{ (\Delta + 18iA p^p \omega) e^{\frac{(9p^p \mu - 18p^p i \omega)t}{18p^p}} - (\Delta - 18iA p^p \omega) e^{\frac{(9p^p \mu + 18p^p i \omega)t}{18p^p}} \right\} \quad (3)$$

Parameters Eq.(3) are:

$$\Delta = 9Ap^p \mu - 18Bp^p \quad (4)$$

And vibration Frequency as follows:

$$\omega = \frac{1}{18p^p} \left(\sqrt{10\alpha^3 p^{2p} - 81p^{2p} \mu^2 - 324\alpha p^{2p} - 648 \beta p^{p+1} + 324 p^p \beta} \right) \quad (5)$$

The following values have been used for the physical parameters of this problem of Eqs.(3-5)as:

$$\alpha := 0.5; \beta := 0.7; \mu := 0.4; A := 0.35; B := 0.2 \quad (6)$$

By substituting the physical values Eq.(6) in the resulting Eqs.(3-5) as:

$$u(t) = 0.35e^{-0.2t} \cos(0.689 t) + 0.4 e^{-0.2x} \sin(0.689 t) \quad (7)$$

$$\omega = 0.689 R / \text{sec} \quad (8)$$

We compare solution of LAM method of Eq.(5) with the numerical method (Runge-Kutte 4th), as shown below:

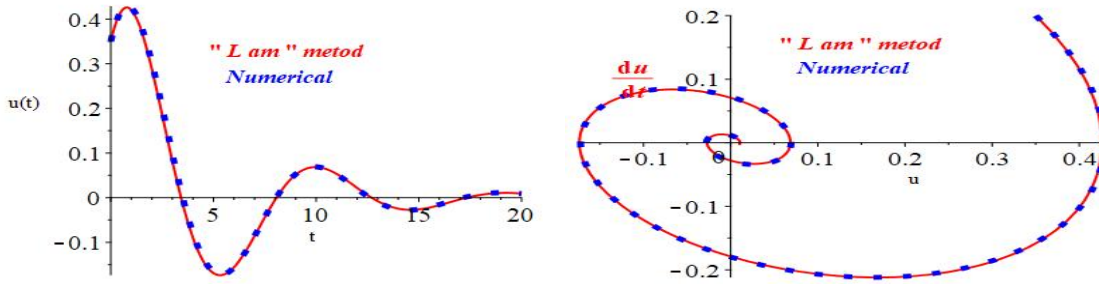


Figure 1: Comparison Between LAM and Numerical solution.

Example 2

We Consider a Vibration Nonlinear Differential Equation as A Nonlinear Phenomenon in The Engineering Field and Basic Sciences As Follows:

$$\ddot{u} + \sin(\alpha u) + \beta u^p + \mu \sin(\Omega x) = 0 \quad (1)$$

The boundary and initial conditions are:

$$u(0) = A, u'(0) = B \quad (2)$$

LAM solution process (Linearize Akbari Method)

Output of the solution process by new approach LAM (Linearize Akbari Method) for nonlinear differential equation Eq.(1) and according to the boundaries and initials conditions Eq.(2) ,the solution of the differential equation is obtained as follows:

$$u(t) = \frac{1}{\omega \psi} (B \psi + 162\mu \Omega p^p) \sin(\omega t) + A \cos(\omega t) - \frac{162}{\psi} p^p \mu \sin(\Omega t) \quad (3)$$

Parameters Eq.(3) are:

$$\psi = -5\alpha^3 p^p + 162\Omega^2 p^p + 162\alpha p^p + 324\beta p - 162\beta \quad (4)$$

And vibration Frequency as follows:

$$\omega = \frac{1}{18} \sqrt{10\alpha^3 - 324\alpha - 648\beta p^{-p+1} + 324\beta p^{-p}} \quad (5)$$

The following values have been used for the physical parameters of this problem of Eqs.(3-5)as:

$$\alpha := -0.5; \beta := -0.1; \mu := -0.6; \Omega := 3.2; A := 0.15; B := 0.1; p := 4 \quad (6)$$

By Substituting the Physical Values Eq.(6) in the resulting Eqs.(3-5) as:

$$u(t) = -0.13748 \sin(0.7063 x) + 0.15 \cos(0.7063 x) + 0.0616 \sin(3.2 x) \quad (7)$$

$$\omega = 0.7063 R / \text{sec} \quad (8)$$

By Substituting the Physical Values Eq.(6) in the resulting Eqs.(3-5) as:

$$u(t) = -0.13748 \sin(0.7063 x) + 0.15 \cos(0.7063 x) + 0.0616 \sin(3.2 x) \quad (7)$$

$$\omega = 0.7063 R / \text{sec} \quad (8)$$

We Compare Solution of LAM Method of Eq.(5) With the Numerical Method (Runge-Kutte 4th) , as shown below:

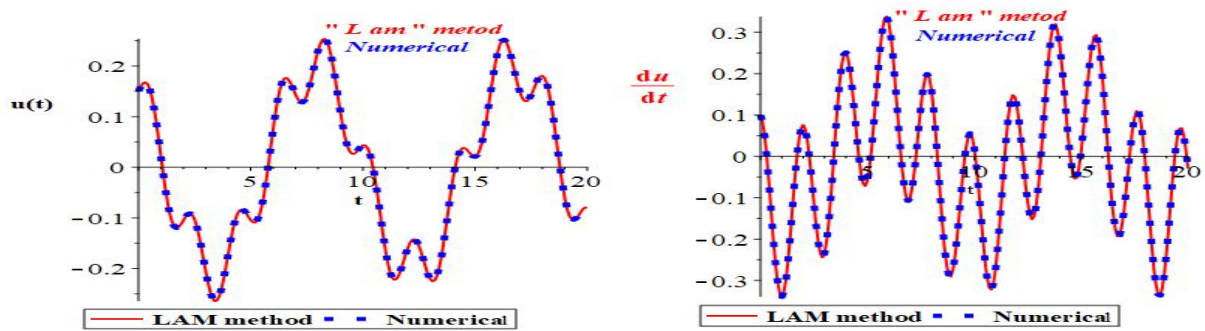


Figure 2: Comparison Between LAM and Numerical Solution.

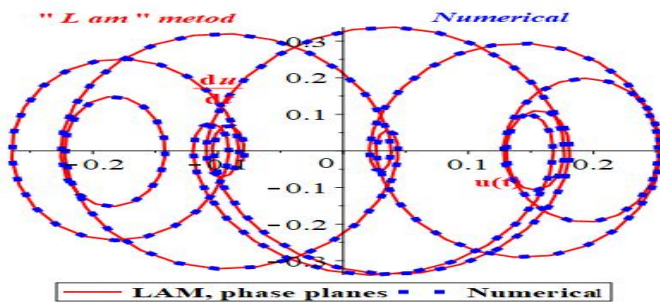


Figure 3: Comparison Between LAM and Numerical Solution for Phase Plans .

iv) Analytical Solution of Complicated Nonlinear Differential Equations (Ode) by Mohammadreza Akbari's Innovative Creative Methods

Example 3

We Consider a Complicated Nonlinear Differential Equation as a Nonlinear Phenomenon in the Engineering Field and Basic Sciences as Follows:

$$u''' = \varepsilon u^2 \sin(\beta u') \quad (1)$$

The Boundary Conditions Are

$$u(0) = u1, u(L) = u2, u'(0) = u3 \quad (2)$$

We analytically solve this non-linear differential equation with ten innovative methods of Mohammad Reza Akbari and measure the output of these methods by comparing the numerical method.

1. AGM Solution Process (Akbari Ganji Method)

Output of the solution process by approach **AGM** for nonlinear differential equation Eq.(1) and according to the boundary conditions Eq.(2) as follows: of the differential equation is obtained as follows:

$$u(x) = -\frac{\varepsilon x^4 \sin\beta}{24L} + \frac{\varepsilon x^3 \sin\beta}{6} - \frac{1}{8L^2} (L^3 \varepsilon \sin\beta + 8L + 8)x^2 + x + 1 \quad (3)$$

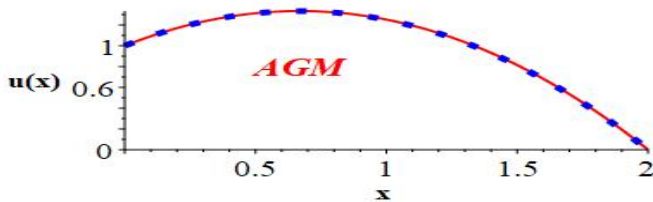


Figure 4: A Comparison Between AGM and Numerical Solution.

2. ASM solution process (Akbari Sara Method)

Output of the solution process by approach **ASM** for nonlinear differential equation Eq.(1) and according to the boundary conditions Eq.(2) as follows: of the differential equation is obtained as follows:

$$u(x) = \frac{\varepsilon}{6} u1^2 \sin(\beta u3)x^3 - \frac{1}{6L^2} (\varepsilon u1^2 \sin(\beta u3)L^3 + 6Lu3 + 6u1 - 6u2)x^2 + u3x + u1 \quad (4)$$

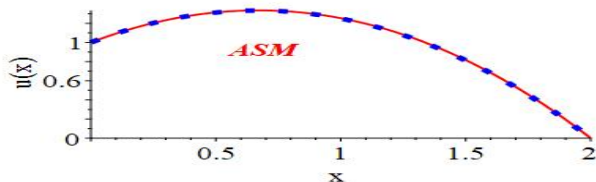


Figure 5: A Comparison Between ASM and Numerical Solution.

3. AYM Solution Process (Akbari Yasna Method)

Output of the solution process by approach **AYM** for nonlinear differential equation Eq.(1) and according to the boundary conditions Eq.(2) as follows:of the differential equation is obtained as follows:

$$u(x) = u1 + u3x + \frac{1}{2} Yx^2 + \frac{1}{6} \varepsilon u1^2 \sin(\beta u3) x^3$$

$$Y = \frac{-1}{3L^2} \{ 6u3 L + \varepsilon u1^2 \sin(\beta u3) L^3 + 6 u1 - 6 u2 \} \quad (5)$$

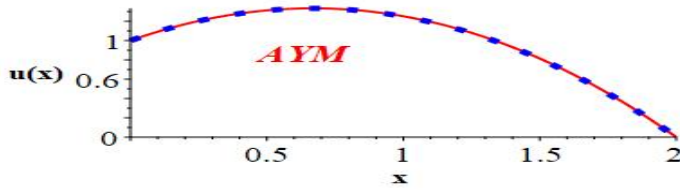


Figure 6: A Comparison Between AYM and Numerical solution.

4. AKLM Solution Process (Akbari Kalantari Leila Method)

Output of the solution process by approach AKLM for nonlinear differential equation Eq.(1) and according to the boundary conditions Eq.(2) as follows: of the differential equation is obtained as follows:

$$u(x) = -\frac{\varepsilon}{6} \sin(\beta)x^3 + \frac{1}{6L^2} (\varepsilon L^3 \sin(\beta) - 6 - 6L) x^2 + x + 1 \quad (6)$$

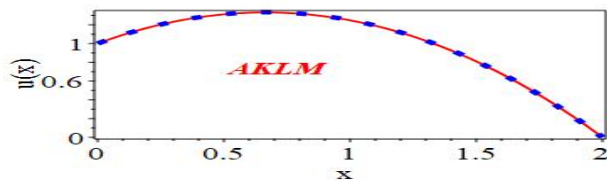


Figure 7: A Comparison Between AKLM and Numerical Solution.

5. MR.AM solution process (Mohammad Reza Akbari Method)

Output of the solution process by approach MR.AM for nonlinear differential equation Eq.(1) and according to the boundary conditions Eq.(2) as follows: of the differential equation is obtained as follows:

$$u(x) = \frac{\varepsilon}{6} \sin(\beta)x^3 - \frac{1}{6L^2} (\varepsilon L^3 \sin\beta) + 6L + 6)x^2 + x + 1 \quad (7)$$

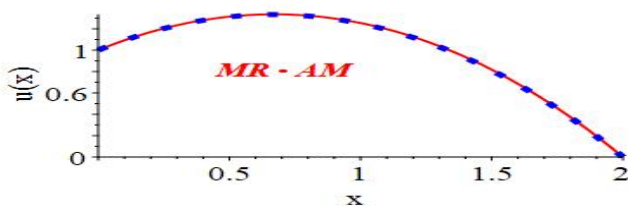


Figure 8: A Comparison Between MR.AM and Numerical Solution.

6. IAM solution process (Integral Akbari Method)

Output of the solution process by approach IAM for nonlinear differential equation Eq.(1) and according to the boundary conditions Eq.(2) as follows: of the differential equation is obtained as follows:

$$u(x) = \frac{1}{4} x^3 \left(\frac{\varepsilon}{2} \sin(\beta) + \frac{\varepsilon}{2} (\Delta + x + 1)^2 \sin(\Delta x + \beta) \right) - \frac{1}{8L^2} (\varepsilon L^3 \sin\beta + 8 + 8L)x^2 + x + 1, \Delta = -(L + 1) \frac{x^2}{L^2} \quad (8)$$

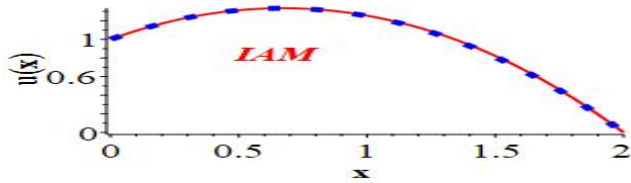


Figure 9: A Comparison Between IAM and Numerical Solution.

7. WoLF-a solution process (Woman life Freedom-akbari Method)

Output of the solution process by approach **WoLF-a** for nonlinear differential equation Eq.(1) and according to the boundary conditions Eq.(2) as follows: of the differential equation is obtained as follows:

$$u(x) = \frac{\varepsilon}{6}x^3 \sin\beta + \frac{1}{24} \left(2\varepsilon^2 L^3(L+2) \sin^2\beta - \frac{2\beta}{L^2}(L+1)\varepsilon \cos\beta \right) x^4 - \frac{1}{12L^2} (12L - L^2\beta\varepsilon(L+1) \cos\beta + 12)x^2 + x + 1 \quad (9)$$

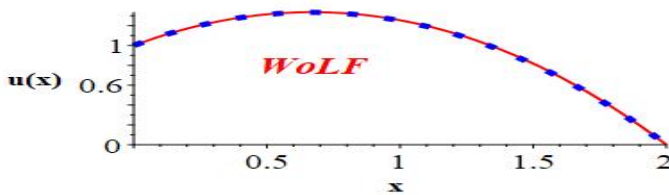


Figure 10: A Comparison Between WoLF-a and Numerical Solution.

8. SYM Solution Process (Sara Yasna Method)

Output of the solution process by approach **SYM** for nonlinear differential equation Eq.(1) and according to the boundary conditions Eq.(2) as follows:of the differential equation is obtained as follows:

$$u(x) = \frac{(L-x)}{480L^2} \left\{ L^2x^2\varepsilon \sin\frac{1}{2}\beta + 4L^2x^2\varepsilon(L^2 + Lx + x^2)\sin\beta - 480Lx - 480L - 480x \right\} \quad (10)$$

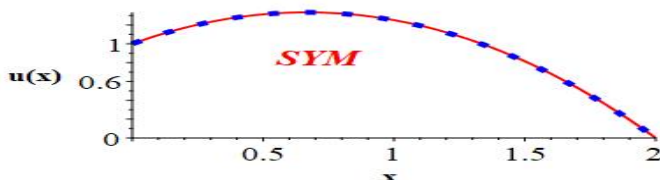


Figure 11: A Comparison Between SYM and Numerical Solution.

9. AxM Solution Process (Akbari x Method)

Output of the solution process by approach **AxM** for nonlinear differential equation Eq.(1) and according to the boundary conditions Eq.(2) as follows: of the differential equation is obtained as follows:

$$u(x) = -\frac{1}{24L^2}(L-x)(x^2\epsilon L^3\sin(\beta) + x^3L^2\epsilon\sin(\beta) - 24Lx - 24L - 24x) \quad (11)$$

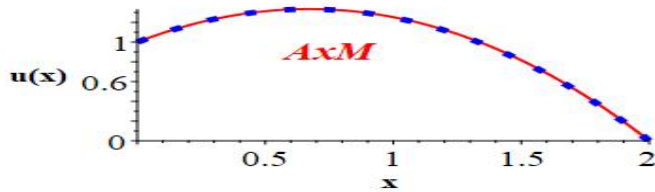


Figure 12: A comparison between AxM and Numerical solution.

10. Mr AM solution process (Mohammadreza Akbari Method)

Output of the solution process by approach Mr AM for nonlinear differential equation Eq.(1) and according to the boundary conditions Eq.(2) as follows: of the differential equation is obtained as follows:

$$u(x) = \frac{\epsilon}{6}\lambda^2\sin\left(\beta\frac{d\lambda}{dx}\right)x^3 - \frac{1}{6L^2}\left(\epsilon L^3\lambda^2\sin\left(\beta\frac{d\lambda}{dx}\right) + 6L + 6\right)x^2 + x + 1, \lambda = 1 + x - \frac{x^2}{L^2}(L + 1) \quad (12)$$

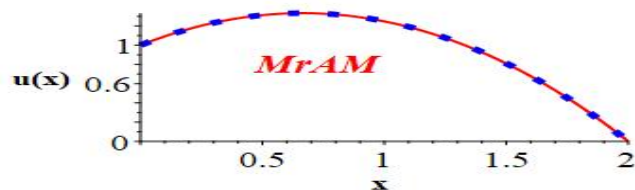


Figure 13: A Comparison Between Mr AM and Numerical solution.

v) General parametric analytical solution of complicated nonlinear differential equations by Mohammadreza Akbari's innovative creative methods

It is noteworthy that until now, no algorithm in the world has been able to solve general nonlinear differential equations in terms of constant coefficients (C1, C2,...) , (without boundary conditions and initial conditions).As a result, one of the "beauty of analytical solution" for nonlinear differentials is the general solution of differential equations (without initial and boundary conditions).One of the biggest problems of old methods (such as HPM, ADM, VIM, DTM) is that they cannot solve equations without boundary conditions. It is an advantage that Mohammadreza Akbari's new creative and innovative methods can answer all kinds of complex nonlinear differential equations as a "general solution".

Example1

Solutions by LAM method

We Consider a Complicated Vibration Nonlinear Differential Equation as Follows:

$$\ddot{u} = \beta \sin(\dot{u}) + \varepsilon u^p \tag{1}$$

General parametric analytical solution by LAM method as:

$$u(t) = C1 e^{\frac{(\psi + 157\beta)t}{324}} + C2 e^{-\frac{(\psi - 157\beta)t}{324}} \tag{2}$$

$$\psi = p^{-p} \sqrt{24649\beta^2 p^{2p} + 209952\varepsilon p^{p+1} - 104976p^p \varepsilon} \tag{3}$$

Here, Eq.(2) is the general solution of non-linear differential Eq.(1), which can be easily calculated by applying the boundary conditions for the constants of C1 and C2.

Applying the initial conditions for computing constant coefficients (C1, C2) as follows:

$$u(0) = A, \dot{u}(0) = B \tag{4}$$

By applying the initial conditions, the coefficients C1, C3 are easily calculated. After calculating and by substituting in Eq.(2), the analytical answer of the differential equation is obtained.

The Following Values Have Been Used for The Physical Parameters in The Dynamical Vibration As

$$A := 0.7; B := 0.2; \varepsilon := -0.5; \beta := -0.4; p = 1 \tag{5}$$

We can compare solution of LAM method of Eqs.(2-4) with the numerical method (Runge-Kutte 4th) as shown below:

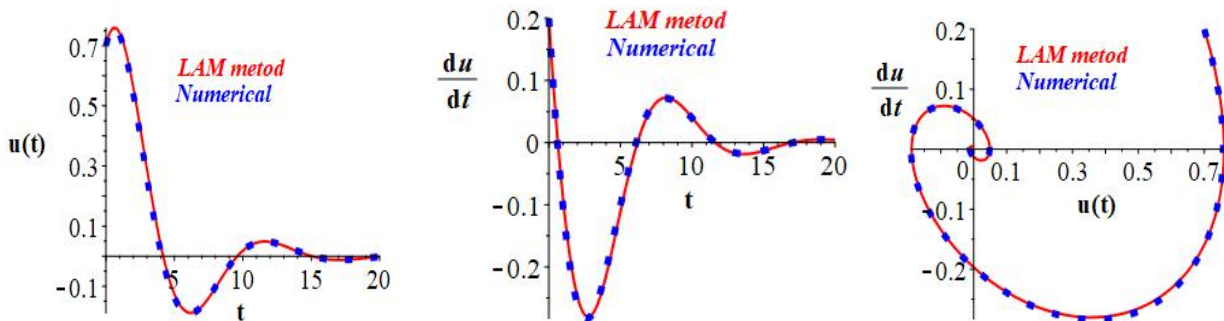


Figure 1: A comparison between LAM and Numerical solution

Example2

Solutions by LAM and method

We consider a complicated **nonlinear differential equation** as follows:

$$u'' = \alpha u^2 u'^3 + \beta u^p \quad (1)$$

General parametric analytical solution by **LAM** method as:

$$u(x) = C1 e^{\frac{(\psi + \alpha p^p)x}{6750 p^p}} + C2 e^{-\frac{(\psi - \alpha p^p)x}{6750 p^p}} \quad (2)$$

$$\psi = \sqrt{\alpha^2 p^{2p} + 10935\alpha p^{2p} + 91125000\beta p^{p+1} - 45562500\beta p^p} \quad (3)$$

Here, Eq.(2) is the general solution of non-linear differential Eq.(1), which can be easily calculated by applying the boundary conditions for the constants of **C1** and **C2**. Applying the initial conditions for computing constant coefficients (**C1**, and **C2**) as follows:

$$u(0) = u1, u(L) = u2 \quad (4)$$

By applying the initial conditions, the coefficients **C1**, and **C2** are easily calculated. After calculating and by substituting in Eq.(2), the analytical answer of the differential equation is obtained.

The Following Values Have Been Used for The Physical Parameters As:

$$L := 8; \alpha := 0.2; \beta := 0.5; u1 := 1; u2 := 0; p = 1 \quad (5)$$

After calculating the constant coefficients, we can compare of solution of **LAM** method of Eqs.(2) and according to Eqs.(3,5) with the numerical method (Runge-Kutte 4th) as shown below:

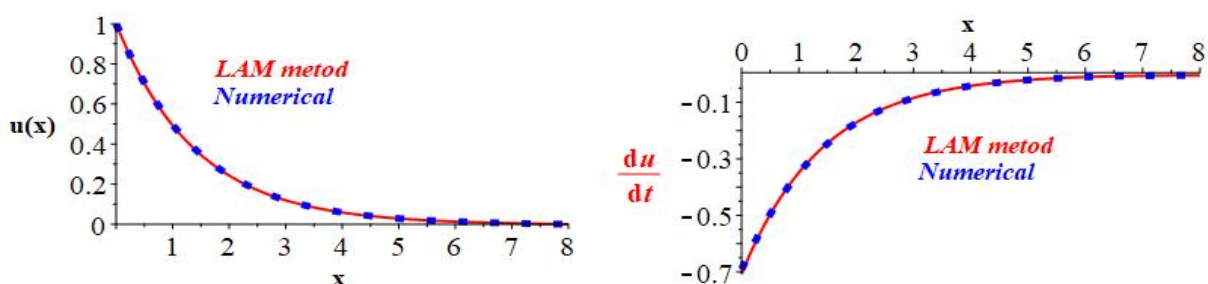


Figure 2: A Comparison Between Lam Methods and Numerical Solution.

Example3

Solutions by Mr AM Method

We Consider a Complicated Partial Nonlinear Differential Equation as Follows:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - \beta u - \eta \frac{\partial u}{\partial x} - \alpha u^2 \quad (1)$$

General Analytical Solution of Complicated Partial Differential Equation by Mr Am Method As:

$$u(x, t) = - \frac{5x^2 e^{-\frac{10(\beta L + 3\eta)t}{3L^3}} (\beta L + 3\eta)}{3\alpha L^3 e^{-\frac{10(\beta L + 3\eta)t}{3L^3}} - 10LCo\beta - 30Co\eta} \times \left\{ \beta (C_2 x + C_1) + \eta C_2 + \alpha (C_2 x + C_1)^2 \right\} + C_2 x + C_1 \quad (2)$$

Here, Eq.(2) is the general solution of partial non-linear differential Eq.(1), which can be easily calculated by applying the boundary conditions for the constants of C1, C2 and C0.

Example4

Solutions by AGM Method

We consider a complicated set of **nonlinear differential equation** as follows:

$$\begin{cases} u_{xx} = \alpha v^2 \sqrt{\sin(uu_x)} \\ v_{xx} = \beta \sqrt{\cos(u^2 + v^2)} \end{cases} \quad (1)$$

Strategy of General Parametric Analytical Solution by **AGM** Method as:

$$u(x) = \frac{\alpha A_1 x^3}{6p \sin(C_1 C_2)} \sqrt{\sin(C_1 C_2)} \left\{ \cos(C_1 C_2) \times \sqrt{\sin(C_1 C_2)} \alpha A_1^3 C_1 + \cos(C_1 C_2) A_1 C_2^2 + 2 \sin(C_1 C_2) p A_2 \right\} + \frac{1}{2} x^2 \alpha A_1^2 \sqrt{\sin(C_1 C_2)} + x C_2 + C_1 \quad (2)$$

$$v(x) = - \frac{x^3}{3q \cos(A_1^2 + C_1^2)} \beta \sqrt{\cos(A_1^2 + C_1^2)} \times \left(A_1 A_2 + C_1 C_2 \right) \sin(A_1^2 + C_1^2) + \frac{1}{2} x^2 \beta \sqrt{\cos(A_1^2 + C_1^2)} + x A_2 + A_1 \quad (3)$$

Here, Eqs.(2,3) is the general solution of non-linear differential Eq.(1), we can easily calculate by applying the boundary conditions for the constants of C1,C2 and A1 and A2.

Example5

Solutions by Mr AM and AGM methods

We Consider a Complicated **Nonlinear Differential Equation** as Follows

$$u'''' = \cos(\beta u) \quad (1)$$

General Parametric Analytical Solution by Mram and Agm Method As

$$u_{AGM} = -\frac{\beta}{120} C_2 \Psi(1 + C_1) C_1! \sin(\beta C_1!) x^5 + \frac{1}{24} \cos(\beta C_1!) x^4 + C_4 x^3 + C_3 x^2 + C_2 x + C_1 \quad (2)$$

$$u_{Mram} = \frac{x^4}{24} \cos\{\beta (C_1 x^3 + C_2 x^2 + C_3 x + C_4)!\} + C_1 x^3 + C_2 x^2 + C_3 x + C_4 \quad (3)$$

Here, Eqs.(2,3) is the general solution of non-linear differential Eq.(1), we can calculate by applying the boundary conditions for the constants of C1,C2,C3 and C4.

vi) Analytical solution of wave differential equations

Example1

Solutions by ‘I am’ method (Integral akbari method) methods

We Consider a Complicated Wave Partial Nonlinear Differential Equation as Follows

$$\frac{\partial u}{\partial t} = \cos\left\{\alpha \sin(u^m t) \cos\left(\frac{\partial^4 u}{\partial x^4}\right)\right\} \quad (1)$$

The initial conditions as:

$$u(x, 0) = \sin(\beta x) \quad (2)$$

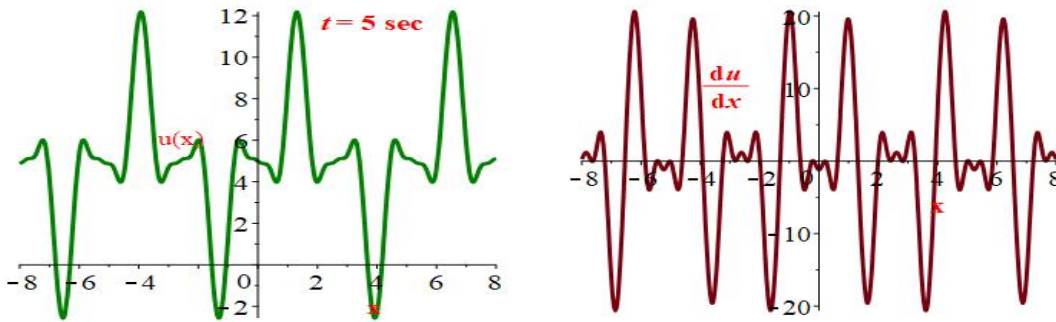
Output of the solution process by new approach ‘I am’ (Integral akbari method) for wave nonlinear differential equation Eq.(1) and according to the initial’s conditions Eq.(2), the solution of the differential equation is obtained as follows:

$$u(x, t) = \frac{1}{4} t^4 \beta^4 \alpha^2 (\sin^{2m} \beta x) \cos(\beta^4 \sin \beta x) \sin(\beta^4 \sin \beta x) - \frac{t^4}{4 \sin(\beta x)} \sin^{2m}(\beta x) \cos(\beta^4 \sin^2(\beta x)) \alpha^2 m - \frac{1}{6} t^3 \alpha^2 \sin^{2m}(\beta x) \cos(\beta^4 \sin^2(\beta x)) \sin(\beta x) + t \quad (2)$$

The Following We Select Physical Values for Parameters as Follows

$$\alpha := 0.2; m := 2, \beta := -1.2 \quad (3)$$

The output diagram of the ‘i am’ method of eq.(2) and according to the physical values eq.(3) is drawn as follows



Figs.1. Graphs of ‘I am’ method for time t=5 sec.

Example2

We Consider a Complicated Wave Partial Nonlinear Differential Equation As Follows:

$$\frac{\partial u}{\partial t} = \cos \left(\alpha u^m t \frac{\partial^2 u}{\partial x^2} \right) \quad (1)$$

The initial conditions as:

$$u(x, 0) = \sin(\beta x) \quad (2)$$

Output of the solution process by new approach ‘I am’ (Integral akbari method) for wave nonlinear differential equation Eq.(1) and according to the initial’s conditions Eq.(2), the solution of the differential equation is obtained as follows:

$$u(x, t) = -\frac{1}{12} \alpha^2 \beta^4 \sin^{2m+1}(\beta x) (3t m + 2 \sin(\beta x) + 3t) t^3 + \sin(\beta x) + t \quad (2)$$

The following we select physical values for parameters as follows:

$$\alpha := 0.2; m := 2, \beta := -1.2 \quad (3)$$

The output diagram of the ‘I am’ method of Eq.(2) and according to the physical values Eq.(3) is drawn as follows:

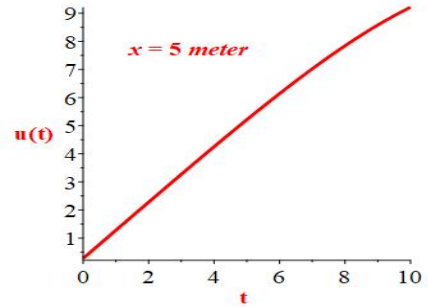
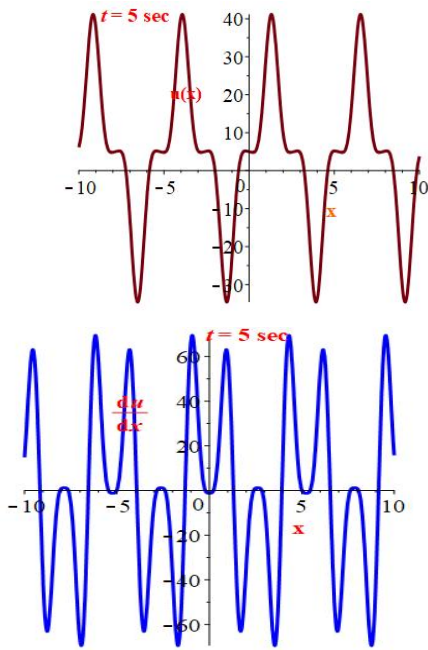


Figure 2: Graphs of ‘I am’ method for time t=5 sec and x=5 meter.

Example3

We consider like before a complicated **wave partial nonlinear differential equation** as follows:

$$\frac{\partial^2 u}{\partial t^2} = m \sqrt{u^2} \frac{\partial^3 u}{\partial x^3} \cos(\beta x) \tag{1}$$

The initial conditions as:

$$u(x, 0) = \sin(\alpha x) ; u_x(x, 0) = 0 \tag{2}$$

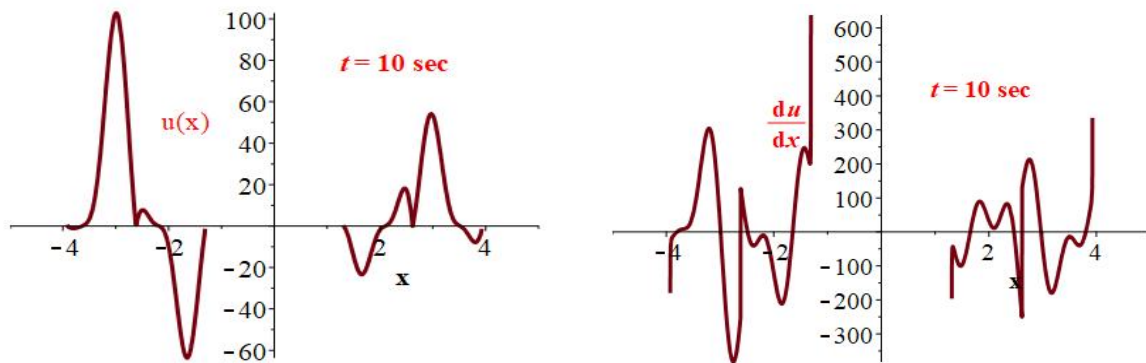
Output of the solution process by new approach ‘I am’ (**Integral akbari method**) for wave nonlinear differential equation Eq.(1) and according to the initial’s conditions Eq.(2), the solution of the differential equation is obtained as follows:

$$\begin{aligned}
 u(x, t) = & \sin(\alpha x) \\
 & - \frac{(m + 4) \left(\sqrt[3]{\alpha^3 \sin(\alpha x) \sin(2\alpha x)} \right)^3 \cos^3(\beta x) t^6}{80m^2 \sin(\alpha x)^2} \\
 & + \frac{\left(\sqrt[3]{\alpha^3 \sin(\alpha x) \sin(2\alpha x)} \right)^2 \cos^2(\beta x) t^4}{8m \sin(\alpha x)} \\
 & + \frac{1}{2} \sqrt[3]{\alpha^3 \sin(\alpha x) \sin(2\alpha x)} \cos(\beta x) t^2 + \dots
 \end{aligned} \tag{2}$$

The following we select physical values for parameters as follows:

$$\alpha := 1.2; \beta := 2.2; m := 2 \tag{3}$$

The output diagram of the “I am” method of Eq.(2) and according to the physical values Eq.(3) is drawn as follows:



Figs.3.Graphs of “I am” method for time t=10 sec .

vii) Analytical solution of Integrals

In this section, we solve some examples of complicated applied integrals with the innovations of Mohammadreza Akbari's new method “I am” method (**Integral akbari method**), and then we compare the output of the analytical solution with the numerical one.

Example1

We consider a complicated **integral equation** as follows:

$$I = \int \sin \left(\{ \alpha x! \}^{\beta x!} \right) x \quad ; \quad x \in (0, \infty) \quad (1)$$

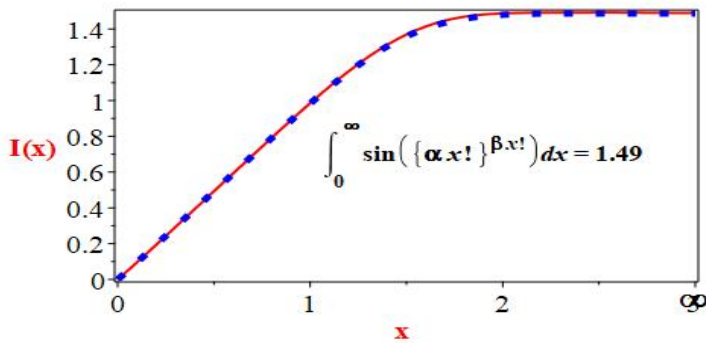
Output of the solution process by new approach “I am” (**Integral akbari method**) for integral equation Eq.(1), the solution of the integral is obtained as follows:

$$I(x) = \frac{x}{4} \left\{ \sin \left(\alpha \left(\frac{x}{8} \right)!^{\beta \left(\frac{x}{8} \right)!} \right) + \sin \left(\alpha \left(\frac{3x}{8} \right)!^{\beta \left(\frac{3x}{8} \right)!} \right) \right. \\ \left. + \sin \left(\alpha \left(\frac{5x}{8} \right)!^{\beta \left(\frac{5x}{8} \right)!} \right) + \sin \left(\alpha \left(\frac{7x}{8} \right)!^{\beta \left(\frac{7x}{8} \right)!} \right) \right\} \quad (2)$$

The following we select physical values for parameters as follows:

$$\alpha := 1.2; \beta := -2.2 \quad (3)$$

The output diagram of the “I am” method of Eq.(2) and according to the physical values Eq.(3) is drawn as follows



Figs.1.Graphs of ‘I am’ method for domain $x \in [0, \infty)$.

Example2

We consider an integral in the coordinates of noncomplex numbers on the closed surface of a circle with radius R, the gg method is able to analytically solve this type of complex integrals, the following integral:

$$I = \oint_s \sin(\varepsilon e^{z^\beta}) dz \tag{1}$$

That s is limited in the circle with the radius R and the center of the coordinate axis as:

$$|z| = R \tag{2}$$

According to the trigonometric equations in polar coordinates as follows:

$$z = x + iy = Re^{i\theta} \tag{3}$$

By substituting Eq.(3) and applying mathematical operations, the integral of Eq.(1) becomes as follows:

$$I = iR \int_0^{2\pi} e^{i\theta} \sin(\varepsilon e^{R^\beta e^{\beta i\theta}}) d\theta \tag{4}$$

At first, we solve the integral analytically without the following integral interval:

$$I = iR \int e^{i\theta} \sin(\varepsilon e^{R^\beta e^{\beta i\theta}}) d\theta \tag{5}$$

Output of the Analytical solution process by new approach ‘I am’ (Integral akbari method) for integral Eq.(5), the solution of the integral is obtained as follows:

$$\begin{aligned}
 I(\theta) = & \frac{R}{100} \ln(\lambda) \left\{ \sin(\varepsilon e^{R^\beta}) - \frac{49Z}{R} \sin(\varepsilon e^{Z^\beta}) \right. \\
 & + 2\sin(\varepsilon e^{(R\lambda^{\beta/50})}) (\lambda)^{1/50} \\
 & + 2\lambda^{2/50} \sin(\varepsilon e^{(R(\lambda)^{2\beta/50})}) + \dots \\
 & \left. + 2\lambda^{49/50} \sin(\varepsilon e^{(R\lambda^{49\beta/50})}) \right\} + \frac{Z}{2} \ln(\lambda) \sin(\varepsilon e^{Z^\beta})
 \end{aligned} \tag{6}$$

Here, we have for parameter in the Eq.(6) as:

$$\lambda = (Z/R), Z = Re^{i\theta} \tag{7}$$

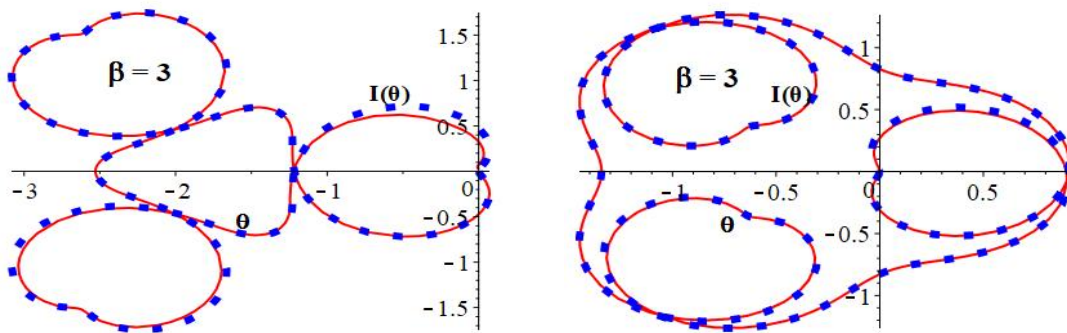
The following we select physical values for parameters as follows:

$$\beta = 3, \varepsilon = 2 \tag{8}$$

Integral value according to Eq.(4) as:

$$I_{IAM} = 0.0, I_{Numerial} = 0.0 \tag{9}$$

The output figures of ‘‘I am’’ method of Eq.(6) and according to the physical values Eq.(6) : different values β are drawn as follows:



Figs.2.Graphs of ‘‘I am’’ method for domain $\theta \in [0, 2\pi)$.

Example3

We consider a complicated **integral equation** as follows:

$$I = \int \sin \left\{ \frac{\alpha \sin(\beta x^3)}{x} \right\} dx; I(0) = 1; x \in (-\infty, \infty) \tag{1}$$

Output of the solution process by new approach ‘‘I am’’ (**Integral akbari method**) for integral equation Eq.(1), the solution of the integral is obtained as follows:

$$I(x) = 1 + \frac{x}{21} \sum_{k=0}^{20} \sin \left\{ \frac{42}{(2k+1)x} \alpha \sin \left(\beta \left(\frac{(2k+1)x}{42} \right)^p \right) \right\} \tag{2}$$

The following we select physical values for parameters as follows:

$$\alpha := 0.5; \beta := 3 \tag{3}$$

The output diagram of the ‘‘I am’’ method of Eq.(2) and according to the physical values Eq.(3) is drawn as follows:

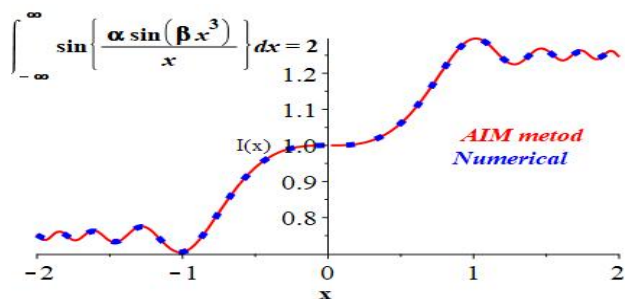


Figure 2: Graphs of “I am” method for domain $x \in [-\infty, \infty)$.

Example 4

We consider a complicated **integral equation** as follows:

$$I = \int \sin \{ \beta \cos(\alpha x)^q \}^p dx ; \quad I(0) = 1 ; \quad x \in (0, 10) \quad (1)$$

Output of the solution process by new approach “I am” (**Integral akbari method**) for integral equation Eq.(1), the solution of the integral is obtained as follows:

$$I(x) = 1 + \frac{x}{201} \sum_{k=0}^{200} \sin^p \left\{ \beta \cos^q \left(\frac{\alpha(2k+1)x}{402} \right) \right\} \quad (2)$$

The following we select physical values for parameters as follows:

$$\alpha := 3; \beta := 5 \quad (3)$$

Equation Eq.(2) is from the output of integral Eq.(1), which is solved by "I am" method, below the graphs of Eq.(2) are drawn according to different values of **p** and **q** according to physical values Eq(3) as:

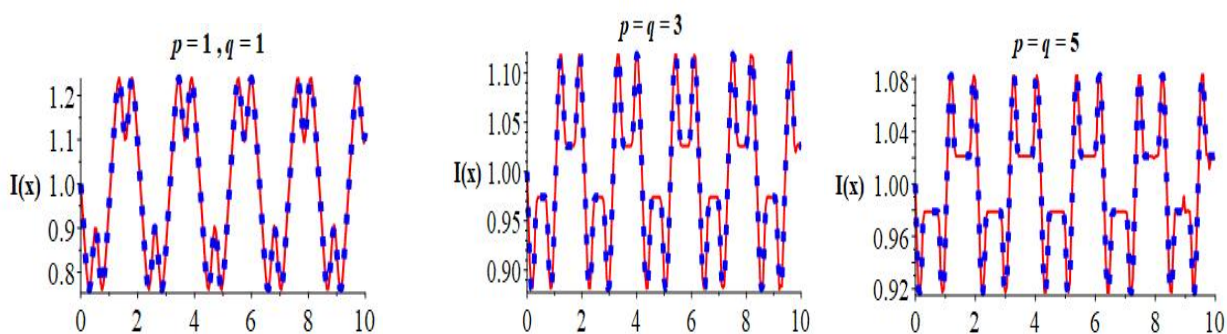


Figure 3: Graphs of “I Am” Method for Domain $x \in [0,10)$.

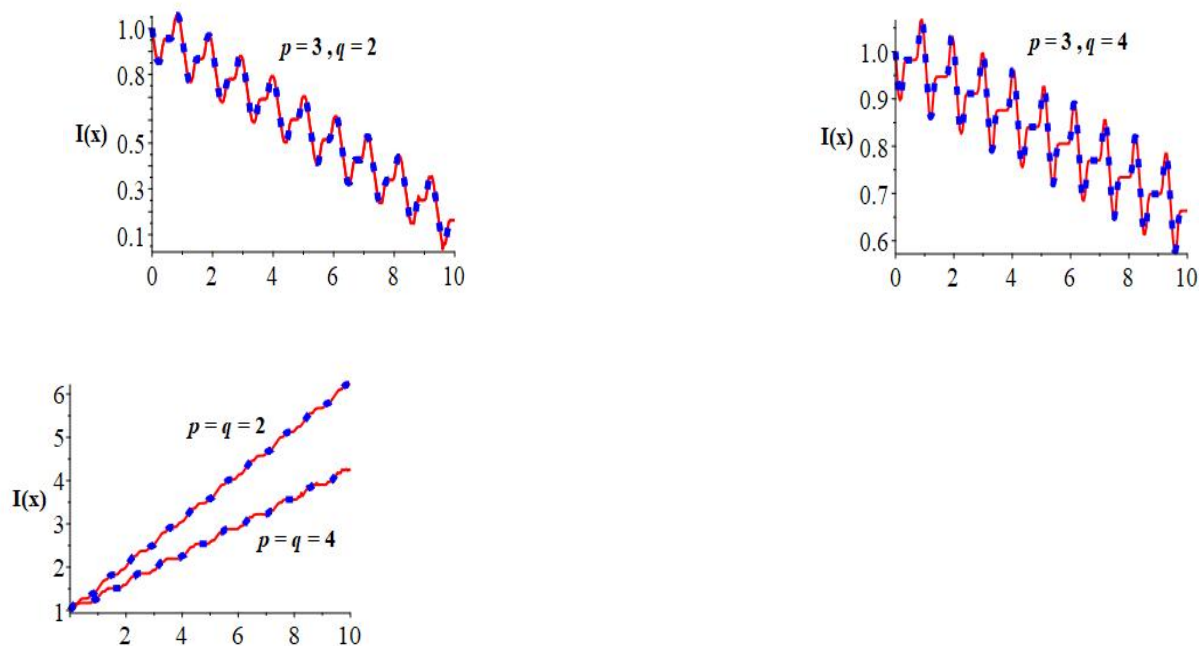


Figure 4: Graphs of ‘I Am’ Method for Domain $x \in [0,10)$.

Conclusions

In this article, we have proven that the creative innovations of Mr. Mohammad Reza Akbari's new methods can very powerfully analyze any type of complex nonlinear differential equations and also integrals. We have been able to challenge many difficult problems in engineering and physics, which many scientists have had difficulty solving or could not solve many nonlinear problems analytically.

Acknowledge, History of the methods invented by M.r.Akbari

*AGM method Akbari Ganji method has been invented mainly by Mohammadreza Akbari in 2014.

*ASM method (Akbari Sara's Method) has been created by Mohammadreza Akbari, in 2019.

*AYM method (Akbari Yasna's Method) has been created by Mohammadreza Akbari, in 2020.

*AKLM method (Akbari Kalantari Leila Method) has been created by Mohammadreza Akbari, in 2020.

*IAM (I am) method (Integral Akbari Method) has been created by Mohammadreza Akbari, in 2021.

*WoLF-a method has been created by Mohammadreza Akbari, in 2022.

*SYM method (Sara Yasna Method) has been created by Mohammadreza Akbari, in 2023.

*MrAM method (Mohammadreza Akbari Method) has been created by Mohammadreza Akbari, in 2023.

*AxM method (Akbari x Method) has been created by Mohammadreza Akbari, in 2023.

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