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# A detailed Study about Heat Transfer in Cylinders Along with the Radius in Transient State by AGM

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***Abstract***

*In the current literature, heat transfer along with the radius of a cylinder has been analyzed. The nonlinear partial differential equation governing on the mentioned system has been investigated by a simple and innovative method which we have named it Akbari-Ganji's Method or AGM. It is notable that in this case study, this method has been compounded by Laplace transform theorem in order to covert the partial differential equation governing on the afore-mentioned system to an ODE and then the yielded equation has been solved conveniently by this new approach (AGM). Comparisons have been made between the mentioned approach and the Numerical Method (rkf45) to check the precision of the presented method. Furthermore, the obtained result from numerical solution indicated that this approach is very efficient and easy so it can be applied for other kinds of nonlinear equations. One of the most important reasons of selecting the mentioned method for solving differential equations in a wide variety of fields not only in heat transfer science but also in different fields of study such as solid mechanics, fluid mechanics, chemical engineering, etc. in comparison with the other methods is as follows: Obviously, according to the order of differential equations, we need boundary conditions so in the case of the number of boundary conditions is less than the order of the differential equation, this method can create additional new boundary conditions in regard to the own differential equation and its derivatives. Therefore, a solution with high precision will be acquired. With regard to the afore-mentioned explanations, the process of solving nonlinear equation(s) will be very easy and convenient in comparison with the other methods.*

**Keywords:** New Idea, Akbari-Ganji's Method (AGM), Heat Transfer, Transient State, Laplace Transform Theorem.

## Introduction

Undoubtedly, heat transfer phenomenon has a significant role in engineering due to its several applications in industry and its environment performance of propulsion systems such as the design of conventional space and water heating systems, cooling of electronic equipment, design of refrigeration and air-conditioning systems and in many manufacturing processes for instance plume and chemical nuclides dispersion, global warming and so on. In most of the situations, the main goal of analyzing heat transfer problems is to obtain the temperature profile within the system and the rate of heat transfer in certain conditions or alternatively, the required conditions such as dimensions, shape and flow measurement in order to achieve the heat rate or temperature distribution or both. Moreover, a broad range of heat conduction problems encountered in engineering applications involves time as an independent variable. The temperature of a body, in general, varies with time as well as position. Generally speaking, we encounter conducting bodies in a three-dimensional Euclidean space in a suitable set of coordinates($x \in R$ ) and the goal is to predict the evolution of the temperature field for future times ($t > 0$) like in this case study which we have considered the variation of temperature with time in a cylinder. It is better for more information to indicate that the goal of analyzing this category of problems is to determine the variation of the temperature as a function of time and position $T(r,t)$ within the heat conducting body. An overview of the published papers in the field of heat transfer reveals that different authors have utilized miscellaneous numerical and analytical methods for solving nonlinear equations. The solution of this kind of problems especially PDEs is a very time-consuming and difficult task. Therefore, attempts have been made by many authors to solve this category of problems with analytical methods. Consequently, due to conquer this difficulty, in recent years much attention has been devoted to the newly developed manners in order to achieve an approximate solution of nonlinear equations such as: Differential Transformation Method (DTM), Homotropy Perturbation Method [1-7]. But the afore-mentioned methods do not have this ability to gain the solution of the presented problem in high precision. Therefore, these nonlinear equations should be solved by using other approaches. Therefore, these complicated nonlinear equations such as the presented problems in this paper should be solved by utilizing other approaches like AGM [8-15]. And in recent years, other new methods have been invented by Mr. Mohammad Reza Akbari, whose analytical solution accuracy of complex problems is very high, and always convergent, easy to understand and use for users, and very flexible, like methods AKLM (Akbari Kalantari Leila Method) [16]. ASM (Akbari Sara's Method), AYM (Akbari Yasna's Method), IAM (Integral Akbari Method), Wolf-a (Women Life Freedom-Akbari method) [17-23].

**Nomenclature 2**

| | |
|---|---|
| $k$ | Thermal conductivity $(\frac{w}{m.c^0})$ |
| $\rho$ | Density $(kg/m^3)$ |
| $c_p$ | Heat capacity of the cylinder $(\frac{kJ}{kg.c^0})$ |
| $r$ | Radial distance $(m)$ |
| $t$ | Time $(s)$ |
| $T(r,t)$ | Temperature in the cylinder at any radial distance $(r)$ and time $(t)$ |
| $\alpha=\frac{k}{\rho.c_p}$ | Thermal diffusivity $(\frac{m^2}{s})$ |
| $\{L\}$ | Laplace operator |

## The analytical method

Boundary conditions and initial conditions are required for analytical methods of each linear and nonlinear differential equation according to the physic of the problem. Therefore, we can solve every differential equation with any degrees. In order to comprehend the given method in this paper, two differential equations governing on engineering processes will be solved in this new manner.

In accordance with the boundary conditions, the general manner of a differential equation is as follows:
The nonlinear differential equation of p which is a function of u, the parameter u which is a function of x and their derivatives are considered as follows:

$$p_k : f\left(u, u', u'', \ldots, u^{(m)}\right)=0 \quad ; \; u=u\left(x\right) \tag{01}$$

Boundary conditions :

$$\begin{cases} u\left(0\right)=u_0, u'\left(0\right)=u_1, \ldots, u^{(m-1)}\left(0\right)=u_{m-1} \\ u\left(L\right)=u_{L_0}, u'\left(L\right)=u_{L_1}, \ldots, u^{(m-1)}\left(L\right)=u_{L_{m-1}} \end{cases} \tag{02}$$

To solve the first differential equation with respect to the boundary conditions in x=L in Eq.(2), the series of letters in the nth order with constant coefficients which is the answer of the first differential equation is considered as follows:

$$u\left(x\right)=\sum_{i=0}^{n} a_i x^i = a_0 + a_1 x^1 + a_2 x^2 + \ldots + a_n x^n \tag{03}$$

The more precise answer of equation (1), the more choice of series sentences from equation (3). In applied problems, approximately five or six sentences from the series are enough to solve nonlinear differential equations. In the answer of differential equation (3) regarding the series from degree (n), there are (n+1) unknown coefficients that need (n+1) equations to be specified. The boundary conditions of Eq. (2) are used to solve a set of equations which is consisted of (n+1) ones.The boundary conditions are applied on the functions such as follows.

**a) The Application of the boundary conditions for the answer of differential Eq. (3) is in the form:**
If x = 0

$$\begin{cases} u\left(0\right)=a_0=u_0 \\ u'\left(0\right)=a_1=u_1 \\ u''\left(0\right)=a_2=u_2 \\ \vdots \qquad \vdots \quad \vdots \end{cases} \tag{04}$$

And when $x = L$ :

$$\begin{cases} u\left(L\right)=a_0+a_1L+a_2L^2+\ldots+a_nL^n=uL_0 \\ u'\left(L\right)=a_1+2a_2L+3a_3L^2+\ldots+na_nL^{n-1}=uL_1 \\ u''\left(L\right)=2a_2+6a_3L+12a_4L^2+\ldots+n\left(n-1\right)a_nL^{n-2}=uL_{m-1} \\ \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \end{cases} \tag{05}$$

**b) After Substituting Eq. (5) into Eq. (1), the Application of the Boundary Conditions on Differential Eq. (1) is done according to the Following Procedure:**

$$\begin{aligned} &p_0: f\,(u\,(0), u'\,(0), u''\,(0), \ldots, u^{(m)}\,(0)) \\ &p_1: f\,(u\,(L), u'\,(L), u''\,(L), \ldots, u^{(m)}\,(L)) \\ &\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \end{aligned} \tag{06}$$

(6) With regard to the choice of n ; ($n<m$) sentences from Eq. (3) and in order to make a set of equations which is consisted of ($n+1$) equations and ($n+1$) unknowns, we confront with a number of additional nknowns which are indeed the same coefficients of Eq. (3). Therefore, to remove this problem, we should derive m times from Eq. (1) due to the additional unknowns in the afore-mentioned set differential equations and then, this is the time to apply the boundary conditions of Eq. (2) on them.

$$\begin{aligned} &p'_k: f\,(u', u'', u''', \ldots, u^{(m+1)}) \\ &p''_k: f\,(u'', u''', u^{(IV)}, \ldots, u^{(m+2)}) \\ &\vdots \quad \vdots \; \vdots \; \vdots \quad \vdots \end{aligned} \tag{07}$$

**c) Application of the Boundary Conditions on the Derivatives of the Differential Equation P_k In Eq. (7) is done in the form of:**

$$p'_k: \begin{cases} f\,(u'(0), u''(0), u'''(0), \ldots, u^{(m+1)}(0)) \\ f\,(u'(L), u''(L), u'''(L), \ldots, u^{(m+1)}(L)) \end{cases} \tag{08}$$

$$p''_k: \begin{cases} f\,(u''(0), u'''(0), \ldots, u^{(m+2)}(0)) \\ f\,(u''(L), u'''(L), \ldots, u^{(m+2)}(L)) \end{cases} \tag{09}$$

(n+1) equations can be made from Eq. (4) to Eq. (9) so that (n+1) unknown coefficients of Eq. (3) take for example a_0,a_1,a_2… .a_n will be computed. The answer of the nonlinear differential Eq. (1) will be acquired by determining coefficients of Eq. (3).

## Application

Referring to Fig.1, the partial differential equation governing on the cylinder is introduced as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T(r,t)}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T(r,t)}{\partial t} \tag{10}$$

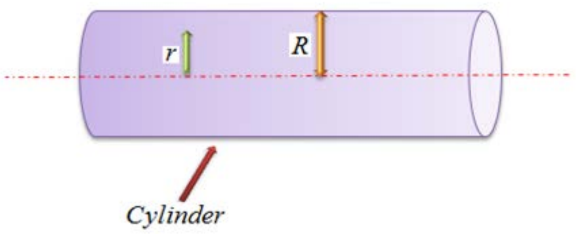


**Figure 1:** The Schematic Diagram of the Physical Model.

The Initial and Boundary Conditions of the Mentioned Cylinder are Expressed In The Forms of:

$$IC: T(r,0) = T_0 \tag{11}$$

$$B.C \begin{cases} T(R,T) = T_1 \\ \dfrac{\partial T}{\partial r}(0,T) = 0 \end{cases} \tag{12}$$

Converting The Partial Differential Equation into Ordinary Differential Equation by Utilizing Laplace Transform Theorem

It is possible to convert the partial differential Eq. (10) into an ordinary differential equation as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T(r,s)}{\partial r}\right) = \frac{1}{\alpha}\{s(T(r,s) - T(\mathrm{r},0)\} \tag{13}$$

In accordance with the given initial conditions in Eq. (11), the afore-mentioned equation will convert to the following ordinary differential equation in terms of  as:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \bar{T}}{\partial r}\right) = \frac{1}{\alpha}(s\bar{T} - T_0) \tag{14}$$

It is notable that the superscript of parameter  shows that Eq. (14) is written in terms of the Laplace variable.

## Solving differential equations with AGM

In AGM approach, the answer of Eq. (10) is considered as a finite series of polynomials with constant coefficients as follows:

$$\bar{T} = \sum_{n=0}^{6} a_n r^n = a_0 + a_1 r + a_2 r^2 + a_3 r^3 + a_4 r^4 + a_5 r^5 + a_6 r^6 \tag{15}$$

## Applying boundary conditions in AGM

The boundary conditions are applied in two ways in AGM:

a) applying the boundary conditions on the answer of the main differential equation which is Eq. (15) is expressed in the form of:

$$\bar{T} = \bar{T}(B.C) \tag{16}$$

It is notable that B.C is the abbreviation of boundary conditions. In this step, the above procedure is also applied in the forms of:

$$T(R) = T_1 \quad \text{therefore} \quad \bar{T}(R) = \frac{T_1}{s}$$
$$a_0 + a_1 R + a_2 R^2 + a_3 R^3 + a_4 R^4 + a_5 R^5 + a_6 R^6 = \frac{T_1}{s} \tag{17}$$

And then:

$$\bar{T}'(R) = 0 \quad \text{so} \quad a_1 = 0 \tag{18}$$

b) applying the boundary conditions on the main differential equation, which is Eq. (14) and is named *f(r)* , and on its derivatives is done after substituting Eq. (15) into differential equation (14) in the following form:

$$f(\bar{T}(r)) \to f(\bar{T}(B.C)) = 0 \quad , \quad f'(\bar{T}(B.C)) = 0 \tag{19}$$

Again, we mention that in accordance with Eq. (16) and Eq. (19), the constant coefficients of Eq. (15) which are $a_0$ to $a_6$ will be obtained very easily. In this step in regard to the above explanations, the application of boundary conditions on the obtained equation from substituting Eq. (15) into Eq. (14) is done as follows:

$$f(R):$$
$$a_1 + 2a_2R + 3a_3R^2 + 4a_4R^3 + 5a_5R^4 + 6a_6R^5 + R(2a_2 + 6a_3R + 12a_4R^2 + 20a_5R^3 + 30a_6R^4) = \frac{R}{\alpha}\{s(a_0 + a_1R + a_2R^2 + a_3R^3 + a_4R^4 + a_5R^5 + a_6R^6) - T_0\} \quad (20)$$

Now this is the best time to apply boundary conditions on the derivatives of the yielded equation from substituting Eq. (15) into Eq. (14) as follows:

$$f'(0): \qquad 4a_2 = \frac{1}{\alpha}(sa_0 - T_0) \quad (21)$$

Then:

$$f'(R):$$
$$4a_2 + 12a_3R + 24a_4R^2 + 40a_5R^3 + 60a_6R^4 + R(6a_3 + 24a_4R + 60a_5R^2 + 120a_6R^3) = \frac{s}{\alpha}\{(a_0 + a_1R + a_2R^2 + a_3R^3 + a_4R^4 + a_5R^5 + a_6R^6) - T_0\} + \frac{R}{\alpha}(a_1 + 2a_2R + 3a_3R^2. 4a_4R^3 + 5a_5R^4 + 6a_6R^5)s \quad (22)$$

**For The Second Derivative of the Obtained Equation, we have:**

$$f''(0): \qquad 18a_3 = \frac{2a_1}{\alpha}s \quad (23)$$

**Afterwards:**

$$f''(R):$$
$$18a_3 + 72a_4R + 180a_5R^2 + 360a_6R^3 + R(24a_4 + 120a_5R + 360a_6R^2) = \frac{s}{\alpha}\{2(a_1 + 2a_2R + 3a_3R^2 + 4a_4R^3 + 5a_5R^4 + 6a_6R^5) + R(2a_2 + 6a_3R + 12a_4R^2 + 20a_5R^3 + 30a_6R^4)\} \quad (24)$$

By solving a set of equations which is consisted of seven equations with seven unknowns from Eqs. (17,18) and Eqs.(20-24), constant coefficients of Eq.(15) can be easily obtained.To simplify, the following new variables are considered as:

$$\begin{aligned}
\xi &= 13R^4s^2 + 488\alpha R^2s + 2400\alpha^2 \\
\psi_1 &= R^6(T_1 - T_0)s^3 - 3R^4\alpha(7T_1 + 19T_0)s^2 + 48R^2\alpha^2(14T_1 - 7sT_0) - 14400\alpha^3T_1 \\
\psi_2 &= (R^6s^3 - 21R^4\alpha s^2 + 672R^2\alpha^2s - 14400\alpha^3)(T_1 - T_0) \\
\psi_3 &= (R^4s^2 - 7R^2\alpha s + 150\alpha^2)(T_1 - T_0) \\
\psi_4 &= (3R^2s + 25\alpha)(T_1 - T_0)
\end{aligned} \quad (25)$$

**The Constant Coefficients of eq. (15) are Achieved with Regard to the above new Variables:**

$$a_0 = \frac{-\psi_1}{6\alpha\xi s}, \quad a_1 = 0, \quad a_2 = \frac{-\psi_2}{24\alpha^2\xi}, \quad a_3 = 0, \quad a_4 = \frac{\psi_3 s}{4\alpha^2\xi}$$
$$a_5 = -\frac{R(T_1 - T_0)s^3}{3\alpha^2\xi}, \quad a_6 = \frac{\psi_4 s^3}{24\alpha^2\xi} \quad (26)$$

By substituting the obtained constant coefficients from Eq. (26) into Eq. (15), the solution of Eq. (14) will be achieved in terms of Laplace parameter as follows

$$\overline{T}(r,s) = \frac{-\psi_1}{6\alpha\xi s} - \frac{\psi_2}{24\alpha^2\xi} r^2 + \frac{\psi_3 s}{4\alpha^2\xi} r^3 - \frac{R(T_1 - T_0)s^3}{3\alpha^2\xi} r^5 + \frac{\psi_4 s^2}{24\alpha^2\xi} r^6 \qquad 27$$

It is noteworthy that the above equation is written in terms of the variable and parameter of Laplace transform which by taking the inverse Laplace transform , the obtained temperature function from Eq.(27) will be converted into .For simplicity, the following new variables are considered as:

$$\begin{aligned}
&\eta_1(r) = 23023(1139r^5 - 2765Rr^4 + 709R^2r^3 + 709R^3r^2 + 52R^4r - 52R^5) \\
&\eta_2(r) = \sqrt{1771}(4827988r^5 - 12806380Rr^4 + 5737553R^2r^3 + 5737553R^3r^2 \\
&-1675271R^4r - 1675271R^5) \\
&\eta_3(r) = -11558\,\mathrm{r}^5 + 298330Rr^4 - 116723R^2r^3 - 116723R^3r^2 - 3289R^4r - 3289R^5
\end{aligned} \qquad 28$$

$$\omega = \frac{4\sqrt{1771}\alpha}{13R^2} \;\; ; \;\; \lambda = \frac{244\alpha}{13R^2}$$

**As Regards the above Variables and Eq.(27), the Solution of the Presented Problem is Gained as follows:**

$$\begin{aligned}
&T(r,t) = T_1 + \frac{(T_1 - T_0)(R - r)\alpha}{93381288\alpha^2 R^8}\{-299299R^6(R + 3r)(R - r)^2.Dirac(1,t) \\
&+R^4 Dirac(t).\eta_1(r) + 4\eta_2(r).R^2\alpha\sinh(\omega t) + \eta_3(r).\mathrm{R}^2\,\alpha\cosh(\omega t)\}\,e^{-\lambda t}
\end{aligned} \qquad 29$$

It is necessary to mention that in the above equation, is Dirac delta function. By choosing the following physical values, we will have:

$$R = 0.5\,(m)\;,\; \alpha = 0.02\,(m^2/s),\; T_0 = 200\,(℃)\;,\; T_1 = 40\,(℃) \qquad 30$$

**Therefore, the 3-D Chart of Temperature Distribution in The Cartesian Coordinates Is Illustrated as Follows:**

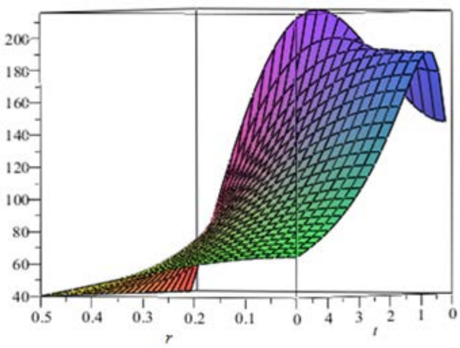


Figure 2: The Obtained 3-D Temperature Profile in Terms of *r* and *t*

Then, a 2-D Contour Can Be Displayed As:

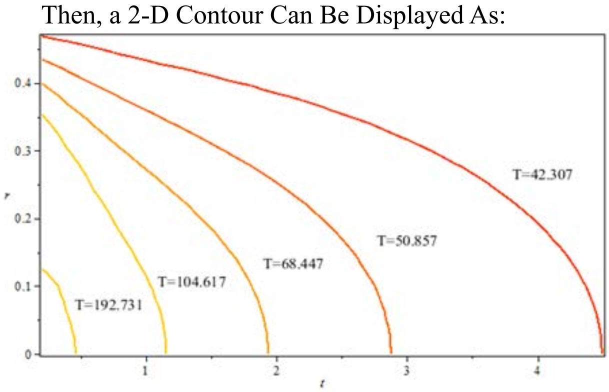


**Figure 3:** The Resulted 2-D Contour for the Temperature Distribution in the Cylinder in Terms of Centigrade.

## Numerical Solution

In regard to the given physical values in Eq.(30) and the achieved temperature profile $T(r,t)$ along with the radius of the cylinder, the achieved results of Numerical Solution in the specified time domain $t \in \{1\,,\,5\}$ in terms of second (s) are presented as follows:

**Table 1: The Acquired Numerical Values for The Temperature Distribution Along with The Radius of The Cylinder According to the Given Physical Values.**

| t \ r | 0 m | 0.1 m | 0.2 m | 0.3 m | 0.4 m | 0.5 m |
|---|---|---|---|---|---|---|
| 1 sec | 186℃ | 181.4℃ | 163℃ | 133.2℃ | 86.38℃ | 40℃ |
| 2 sec | 139.3℃ | 134.6℃ | 119.3℃ | 95.65℃ | 68.3℃ | 40℃ |
| 3 sec | 103.5℃ | 100.75℃ | 90.08℃ | 75.2℃ | 57.16℃ | 40℃ |
| 4 sec | 80℃ | 78.03℃ | 71.5℃ | 62.3℃ | 50.5℃ | 40℃ |
| 5 sec | 65.5℃ | 63.5℃ | 59.01℃ | 53.4℃ | 46.4℃ | 40℃ |

## Comparing the achieved solutions by AGM and Numerical method

The following charts are depicted in five different time steps, $t=$ 1, 2, 3, 4, 5, by AGM and Numerical

Method In Order to Compare the Achieved Solution Together in the Following Forms

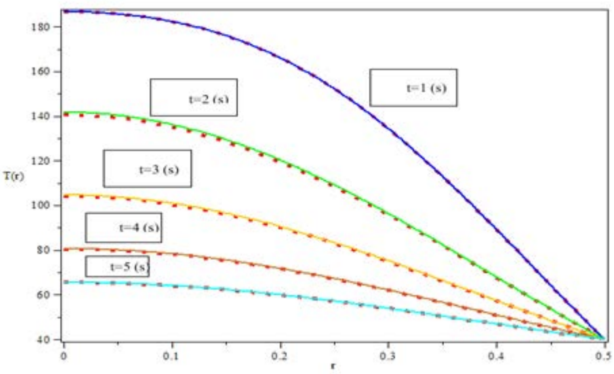


**Figure 4:** A comparison between the obtained solutions by AGM and Numerical Method in . $t=1,2,3,4,5(s)$

## Conclusions

In the present paper, a new idea (AGM) for solving the governing equation of heat transfer along with the radius of a cylinder has been offered. The above procedure has been done to indicate the ability of AGM for solving a wide variety of differential equations because there are significant advantages in solving differential equations by AGM. To understand more, being familiar with the following AGM benefits is recommended such as: Instead of complicated procedures in other analytical methods, by utilizing a very convenient and simple approach, nonlinear differential equations even partial differential equations like in this case study can be answered by solving a set of algebraic equations. Therefore, a broad range of students even with medium mathematical knowledge are able to solve complicated differential equations. With regard to table.1 and the comparisons which have been done between AGM and numerical method, it is clear that the process of solving nonlinear differential equations by AGM is very reliable. Furthermore, in AGM the solution procedure is very short and simple in comparison with the other methods. Eventually, in contrast to the other analytical methods in this approach, there is no need to convert variables of the problem and also boundary conditions into new ones to use dimensionless parameters. In accordance with the mentioned reasons, take for example the shortage of boundary condition(s) for solving differential equation(s) is terminated completely and in order to be able to solve differential equations directly without utilizing dimensionless parameters, AGM will be operational for miscellaneous nonlinear differential equations. Consequently, we are hopeful in the near future this new idea will be applied by the ones who want to challenge complicated nonlinear differential equations.

## History Of Agm , Asm , Aym , Aklm , Mr. Am And Iam, Wolf-A, Sym Methods.

**AGM** (Akbari-Ganji Methods), ASM (Akbari-Sara's Method) , AYM (Akbari-Yasna's Method) AKLM (Akbari Kalantari Leila Method), MR.AM (Mohammadreza Akbari Method)and IAM ( Integral Akbari Methods),WoLF,a method (Women Life Freedom, Akbari), have been invented mainly by Mohammadreza Akbari (M.R.Akbari) in order to provide a good service for researchers who are a pioneer in the field of nonlinear differential equations.

*AGM method Akbari Ganji method has been invented mainly by Mohammadreza Akbari in 2014. Noting that Prof. Davood Dom airy Ganji co-operated in this project.

***ASM** method (Akbari Sara's Method) has been created by Mohammadreza Akbari on 22 of August, in 2019.

***AYM** method (Akbari Yasna's Method) has been created by Mohammadreza Akbari on 12 of April, in 2020.

***AKLM** method (Akbari Kalantari Leila Method)has been created by Mohammadreza Akbari on 22 of August, in 2020.

***MR.AM** method (Mohammadreza Akbari Method)has been created by Mohammadreza Akbari on 10 of November, in 2020.

***IAM** method (Integral Akbari Method)has been created by Mohammadreza Akbari on 5 of February, in 2021.

***Wolf- a** method (Women Life Freedom-Akbari)has been created by Mohammadreza Akbari on 5 of February, in 2022.

***SYM** method (Women Life Freedom, Akbari)has been created by Mohammadreza Akbari on 5 of February, in 2023.

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